Cálculo 1 - Professor Alessandro Monteiro

Lista 02 - Limites

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01. Resolva os Limites:

1. $\lim\limits_{x\to 2} \dfrac{x^2-4}{x-2}$

2. $\lim\limits_{x\to 3} \dfrac{x^2-8x+15}{x^2-2x-3}$

3. $\lim\limits_{x\to 2} \dfrac{\sqrt{x^2+12}-4}{x-2}$

4. $\lim\limits_{x\to 1} \dfrac{\frac{1}{x}-1}{x-1}$

5. $\lim\limits_{h\to 0} \dfrac{\sqrt{4+h}-2}{h}$

6. $\lim\limits_{x\to 2} \dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}$

7. $\lim\limits_{x\to 3} \dfrac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$

8. $\lim\limits_{x\to -1} \dfrac{x^2-3x-4}{x^2+5x+4}$

9. $\lim\limits_{x\to -2} \dfrac{x^2-4}{x^2-x-6}$

10. $\lim\limits_{x\to 2} \dfrac{x^2-7x+10}{x^2+x-6}$

11. $\lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$

12. $\lim\limits_{x\to -1} \dfrac{\frac{1}{x}+1}{x+1}$

13. $\lim\limits_{x\to 4} \dfrac{x-4}{x^2-16}$

14. $\lim\limits_{x\to 3} \dfrac{x^2+7x-30}{x-3}$

15. $\lim\limits_{x\to 0^+} \dfrac{2x}{|x|}$

16. $\lim\limits_{x\to 3^+} \dfrac{3x-9}{|x-3|}$

17. $\lim\limits_{x\to 0^-} \dfrac{|x|}{4x} =$

18. $\lim\limits_{x\to 1^+} \dfrac{2x-2}{|x-1|}$

19. $\lim\limits_{x\to 1^-} \dfrac{2x-2}{|x-1|}$

20. $\lim\limits_{x\to 1} \dfrac{x^2-4x+3}{x^2+4x-5}$

21. $\lim\limits_{h\to 0} \dfrac{\sqrt{5+h}-\sqrt{5}}{h}$

22. $\lim\limits_{h\to 0} \dfrac{\sqrt{7-h}-\sqrt{7}}{h}$

23. $\lim\limits_{h\to 0} \dfrac{\sqrt{5-h}-\sqrt{5}}{h}$

24. $\lim\limits_{x\to 3} \dfrac{\sqrt{x}-\sqrt{3}}{x-3}$

25. $\lim\limits_{h\to 0} \dfrac{\frac{1}{6+h}-\frac{1}{6}}{h}$

26. $\lim\limits_{x\to 8} \dfrac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})}$

27. $\lim\limits_{x\to 2} \dfrac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})}$

28. $\lim\limits_{x\to 1^+} \dfrac{\sqrt{2x}(x-1)}{|x-1|}$

29. $\lim\limits_{x\to 4^+} \dfrac{x^2+2x+8}{|-x+4|}$

30. $\lim\limits_{x\to 3^+} \dfrac{-x^2-2x+15}{|-x+3|}$

31. $\lim\limits_{x\to 3^-} \dfrac{-x^2+x+6}{|-x+3|}$

32. $\lim\limits_{x\to 3^-} \dfrac{-x^2+x+6}{-x+3}$

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02. Calcule os limites:

a) $\lim\limits_{x\to -1} \dfrac{x^3+5x+7}{x^2-6x+8}$ b) $\lim\limits_{x\to 3} \dfrac{x^2-5x+6}{x-3}$ c) $\lim\limits_{x\to 1} \dfrac{x^4-1}{x-1}$

d) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$ e) $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}$ f) $\lim\limits_{x\to 4} \dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$

g) $\lim\limits_{x\to 3} \dfrac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}$ h) $\lim\limits_{x\to a} \dfrac{x^k-a^k}{x-a}$

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03. Encontre os Limites:

Let $f(x)=\begin{cases} 2x-1, & \text{if } x<2; \\ x^2+1, & \text{if } x>2. \end{cases}$ Then $\lim\limits_{x\to 2^-} f(x) =$ ____ and $\lim\limits_{x\to 2^+} f(x) =$ ____ .

Let $f(x)=\dfrac{|x-1|}{x-1}$. Then $\lim\limits_{x\to 1^-} f(x) =$ ____ and $\lim\limits_{x\to 1^+} f(x) =$ ____ .

Let $f(x)=\begin{cases} 5x-3, & \text{if } x<1; \\ x^2, & \text{if } x\geq 1. \end{cases}$ Then $\lim\limits_{x\to 1^-} f(x) =$ ____ and $\lim\limits_{x\to 1^+} f(x) =$ ____ .

Let $f(x)=\begin{cases} 3x+2, & \text{if } x<-2; \\ x^2+3x-1, & \text{if } x\geq -2. \end{cases}$ Then $\lim\limits_{x\to -2^-} f(x) =$ ____ and $\lim\limits_{x\to -2^+} f(x) =$ ____ .

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04. Resolva:

k) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x}$ l) $\lim\limits_{x\to 0} \dfrac{tgx}{x}$

m) $\lim\limits_{x\to 0} \dfrac{x}{senx}$ n) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$ o) $\lim\limits_{x\to 0} x\cdot sen\dfrac{1}{x}$

p) $\lim\limits_{x\to 0} \dfrac{senax}{bx}$ q) $\lim\limits_{x\to 0} \dfrac{senax}{senbx}$ r) $\lim\limits_{x\to 0} \dfrac{senax}{tgbx}$

s) $\lim\limits_{x\to 0} \left(\dfrac{1}{senx}-\dfrac{1}{tgx}\right)$ t) $\lim\limits_{x\to \frac{\pi}{2}} \left(\dfrac{\pi}{2}-x\right)tgx$ u) $\lim\limits_{x\to 1} \dfrac{sen(x-1)}{x^2-1}$

v) $\lim\limits_{n\to \infty} \dfrac{n^2\cdot \cos n!}{n^3+1}$ w) $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$, onde $f(x)=x^2-3x$

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05. Resolva:

$$\lim_{x\to\infty} \frac{(5x^{10}+32)^3}{(1-2x^6)^5} = -\frac{a}{32} \text{ where } a = \_\_\_\_\_\_\_\_\_\_ .$$

$$\lim_{x\to\infty} \left(\sqrt{x^2+x}-x\right) = \frac{1}{a} \text{ where } a = \_\_\_\_\_ .$$

$$\lim_{x\to\infty} x(256x^4+81x^2+49)^{-1/4} = \frac{1}{a} \text{ where } a = \_\_\_\_\_ .$$

$$\lim_{x\to\infty} x\left(\sqrt{3x^2+22}-\sqrt{3x^2+4}\right) = a\sqrt{a} \text{ where } a = \_\_\_\_\_ .$$

$$\lim_{x\to\infty} x^{\frac{2}{3}}\left((x+1)^{\frac{1}{3}}-x^{\frac{1}{3}}\right) = \frac{1}{a} \text{ where } a = \_\_\_\_\_ .$$

$$\lim_{x\to\infty} \left(\sqrt{x+\sqrt{x}}-\sqrt{x-\sqrt{x}}\right) = \_\_\_\_\_ .$$

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06. Calcule os Limites:

a) $\lim\limits_{n\to+\infty} \dfrac{2^n+1}{3^n+2}$ b) $\lim\limits_{x\to+\infty} 2013^x$ c) $\lim\limits_{x\to+\infty} \left(1+\dfrac{1}{x}\right)^x$ d) $\lim\limits_{h\to 0} (1+h)^{\frac{1}{h}}$

e) $\lim\limits_{h\to 0} \dfrac{e^h-1}{h}$ f) $\lim\limits_{x\to+\infty} \left(1+\dfrac{3}{x}\right)^x$ g) $\lim\limits_{x\to+\infty} \left(1+\dfrac{1}{x}\right)^{2013x}$ h) $\lim\limits_{x\to+\infty} \left(1+\dfrac{3}{x}\right)^{5x}$

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07. Mostre que:

a) $\lim\limits_{h\to 0} \dfrac{\ln(1+h)}{h} = 1$ b) $\lim\limits_{h\to 0} \dfrac{\log(1+h)}{h} = \log_a e$ c) $\lim\limits_{h\to 0} \dfrac{a^h-1}{h} = \ln a$

08.

Um aluno escreveu em seu caderno:

Usando a definição de limite, é possível mostrar que $lim_{x\to 1}(3x+2)=5$ *da seguinte forma:*	
1	*Mostremos que, para qualquer* $\epsilon > 0$ *existe* $\delta > 0$ *tal que:*
2	$0 < \lvert x-1\rvert < \delta \Longrightarrow \lvert(3x+2)-5\rvert > \epsilon$
3	*Assim:*
4	$\lvert(3x+2)-5\rvert > \epsilon \Leftrightarrow \lvert 3x-3\rvert > \epsilon \Leftrightarrow 3\lvert x-1\rvert > \epsilon \Leftrightarrow \lvert x-1\rvert > \frac{\epsilon}{3}$
5	*Escolhido* $\delta = \frac{\epsilon}{3}$, *teremos:*
6	$\forall\epsilon > 0, \exists\delta = \frac{\epsilon}{3} > 0 \mid 0 < \lvert x-1\rvert < \delta \Longrightarrow \lvert(3x+2)-5\rvert < \epsilon$
7	*De fato:*
8	$0 < x-1 < \delta = \frac{\epsilon}{3} \Longrightarrow \lvert x-1\rvert < \frac{\epsilon}{3} \Longrightarrow 3x-1 < \epsilon \Longrightarrow$
9	$\Longrightarrow \lvert 3x-3\rvert < \epsilon \Longrightarrow \lvert(3x+2)-5\rvert < \epsilon$

Em dúvida quanto à sua demonstração, tal aluno mostrou-a a seu professor e descobriu que cometera alguns erros. O professor, então, propôs que os dois juntos revisassem e corrigissem os erros. Assim, serão corrigidas, na demonstração, as linhas em que estão presentes os erros cometidos pelo aluno, são elas:

a) 6, 8 e 9 b) 2, 4 e 8 c) 4, 6 e 9 d) 2, 4 e 6 e) 4, 8 e 9

09.

Se $2x \leq g(x) \leq x^4 - x^2 + 2$ para todo x, encontre $\lim_{x\to 1} g(x)$.

10.

a) Find

$\lim_{x\to 1} f(x)$, where $2x-1 \leq f(x) \leq x^2$ for all x in the interval $(0,2)$.

b)

Use the squeeze theorem to show that $\lim_{x\to 0^+}\left(\sqrt{x}e^{\sin(1/x)}\right) = 0$.