Función logarítmica

1. Find the value of y.

   (1) $\log_5 25 = y$ (2) $\log_3 1 = y$ (3) $\log_{16} 4 = y$ (4) $\log_2 \frac{1}{8} = y$

   (5) $\log_5 1 = y$ (6) $\log_2 8 = y$ (7) $\log_7 \frac{1}{7} = y$ (8) $\log_3 \frac{1}{9} = y$

   (9) $\log_y 32 = 5$ (10) $\log_9 y = -\frac{1}{2}$ (11) $\log_4 \frac{1}{8} = y$ (12) $\log_9 \frac{1}{81} = y$

2. Evaluate.

   (1) $\log_3 1$ (2) $\log_4 4$ (3) $\log_7 7^3$ (4) $b^{\log_b 3}$ (3) $\log_{25} 5^3$ (4) $16^{\log_4 8}$

3. Write the following expressions in terms of logs of x, y and z.

   (1) $\log x^2 y$ (2) $\log \frac{x^3 y^2}{z}$ (3) $\log \frac{\sqrt{x}\sqrt[3]{y^2}}{z^4}$ (4) $\log xyz$

   (5) $\log \frac{x}{yz}$ (6) $\log \left(\frac{x}{y}\right)^2$ (7) $\log (xy)^{\frac{1}{3}}$ (8) $\log x\sqrt{z}$

   (9) $\log \frac{\sqrt[3]{x}}{\sqrt[3]{yz}}$ (10) $\log \sqrt[4]{\frac{x^3 y^2}{z^4}}$ (11) $\log x\sqrt{\frac{\sqrt{x}}{z}}$ (12) $\log \sqrt{\frac{xy^2}{z^8}}$

4. Write the following equalities in exponential form.

(1) $\log_3 81 = 4$ (2) $\log_7 7 = 1$ (3) $\log_{\frac{1}{2}} \dfrac{1}{8} = 3$ (4) $\log_3 1 = 0$

(5) $\log_4 \dfrac{1}{64} = -3$ (6) $\log_6 \dfrac{1}{36} = -2$ (7) $\log_x y = z$ (8) $\log_m n = \dfrac{1}{2}$

5. Write the following equalities in logarithmic form.

(1) $8^2 = 64$ (2) $10^3 = 10000$ (3) $4^{-2} = \dfrac{1}{16}$ (4) $3^{-4} = \dfrac{1}{81}$

(5) $\left(\dfrac{1}{2}\right)^{-5} = 32$ (6) $\left(\dfrac{1}{3}\right)^{-3} = 27$ (7) $x^{2z} = y$ (8) $\sqrt{x} = y$

6. True or False?

(1) $\log\left(\dfrac{x}{y^3}\right) = \log x - 3\log y$ (2) $\log(a-b) = \log a - \log b$ (3) $\log x^k = k \cdot \log x$

(4) $(\log a)(\log b) = \log(a+b)$ (5) $\dfrac{\log a}{\log b} = \log(a-b)$ (6) $(\ln a)^k = k \cdot \ln a$

(7) $\log_a a^a = a$ (8) $-\ln\left(\dfrac{1}{x}\right) = \ln x$ (9) $\ln_{\sqrt{x}} x^k = 2k$

7. Solve the following logarithmic equations.

(1) $\ln x = -3$ (2) $\log(3x-2) = 2$

(3) $2\log x = \log 2 + \log(3x-4)$ (4) $\log x + \log(x-1) = \log(4x)$

(5) $\log_3(x+25) - \log_3(x-1) = 3$ (6) $\log_9(x-5) + \log_9(x+3) = 1$

(7) $\log x + \log(x-3) = 1$ (8) $\log_2(x-2) + \log_2(x+1) = 2$

8. Prove the following statements.

   (1) $\log_{\sqrt{b}} x = 2\log_b x$ (2) $\log_{\frac{1}{\sqrt{b}}} \sqrt{x} = -\log_b x$ (3) $\log_{b^4} x^2 = \log_b \sqrt{x}$

9. Given that $\log 2 = x$, $\log 3 = y$ and $\log 7 = z$, express the following expressions in terms of x, y, and z.

   (1) $\log 12$ (2) $\log 200$ (3) $\log \dfrac{14}{3}$ (4) $\log 0.3$

   (5) $\log 1.5$ (6) $\log 10.5$ (7) $\log 15$ (8) $\log \dfrac{6000}{7}$

10. Solve the following equations.

    (1) $3^x - 2 = 12$ (2) $3^{1-x} = 2$

    (3) $4^x = 5^{x+1}$ (4) $6^{1-x} = 10^x$

    (5) $3^{2x+1} = 2^{x-2}$ (6) $\dfrac{10}{1+e^{-x}} = 2$

    (7) $5^{2x} - 5^x - 12 = 0$ (8) $e^{2x} - 2e^x = 15$

11. Draw the graph of each of the following logarithmic functions, and analyze each of them completely.

    (1) $f(x) = \log x$ (2) $f(x) = \log -x$

    (3) $f(x) = -\log(x-3)$ (4) $f(x) = -2\log_3(3-x)$

    (5) $f(x) = -\ln(x+1)$ (6) $f(x) = 2\ln\dfrac{1}{2}(x+3)$

    (7) $f(x) = \ln(2x+4)$ (8) $f(x) = -2\ln(-3x+6)$

12. Find the inverse of each of the following functions.

    (1) $f(x) = \log_2(x-3) - 5$ (2) $f(x) = 3\log_3(x+3) + 1$

    (3) $f(x) = -2\log 2(x-1) + 2$ (4) $f(x) = -\ln(1-2x) + 1$

    (5) $f(x) = 2^x - 3$ (6) $f(x) = 2 \cdot 3^{3x} - 1$

    (7) $f(x) = -5 \cdot e^{-x} + 2$ (8) $f(x) = 1 - 2e^{-2x}$

13. 15 000$ is invested in an account that yeilds 5% interest per year. After how many years will the account be worth 91 221.04$ if the interest is compounded yearly?

14. 8 000$ is invested in an account that yeilds 6% interest per year. After how many years will the account be worth 13709.60$ if the interest is compounded monthly?

15. Starting at the age of 40, an average man loses 5% of his hair every year. At what age should an average man expect to have half his hair left?

16. A bacteria culture starts with 10 00 bacteria and the number doubles every 40 minutes.

    (a) Find a formula for the number of bacteria at time t.

    (b) Find the number of bacteria after one hour.

    (c) After how many minutes will there be 50 000 bacteria?

ANSWERS

1. (1) 2

   (2) 0

   (3) $\frac{1}{2}$

   (4) -3

   (5) 0

   (6) 3

   (7) -1

   (8) -2

   (9) 2

   (10) $\frac{1}{3}$

   (11) $-\frac{3}{2}$

   (12) -2

2. (1) 0

   (2) 1

   (3) 3

   (4) 3

   (5) $\frac{3}{2}$

   (6) 64

3. (1) $2\log x + \log y$

   (2) $3\log x + 2\log y - \log z$

   (3) $\frac{1}{2}\log x + \frac{2}{3}\log y - 4\log z$

   (4) $\log x + \log y + \log z$

   (5) $\log x - \log y - \log z$

   (6) $2\log x - 2\log y$

   (7) $\frac{1}{3}\log x + \frac{1}{3}\log y$

   (8) $\log x + \frac{1}{2}\log z$

   (9) $\frac{1}{3}(\log x - \log y - \log z)$

   (10) $\frac{1}{4}\log x + \frac{1}{2}\log y - \log z$

   (11) $\frac{5}{4}\log x - \frac{1}{2}\log z$

   (12) $\frac{1}{2}\log x + \log y - 4\log z$

4. (1) $3^4 = 81$

   (2) $7^1 = 7$

   (3) $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

   (4) $3^0 = 1$

   (5) $4^{-3} = \frac{1}{64}$

   (6) $6^{-2} = \frac{1}{36}$

   (7) $x^z = y$

   (8) $m^{\frac{1}{2}} = n$

5. (1) $\log_8 64 = 2$

   (2) $\log_{10} 10000 = 3$

   (3) $\log_4 \frac{1}{16} = -2$

   (4) $\log_3 \frac{1}{81} = -4$

   (5) $\log_{\frac{1}{2}} 32 = -5$

   (6) $\log_{\frac{1}{3}} 27 = -3$

   (7) $\log_x y = 2z$

   (8) $\log_x y = \frac{1}{2}$

6. (1) True

   (2) False

   (3) True

   (4) False

   (5) False

   (6) False

   (7) True

   (8) True

7. (1) $S = \{e^{-3}\}$

   (2) $S = \{34\}$

   (3) $S = \{2, 4\}$

   (4) $S = \{5\}$

   (5) $S = \{2\}$

   (6) $S = \{6\}$

   (7) $S = \{5\}$

   (8) $S = \{3\}$

8. (1)

$$\log_{\sqrt{b}} x = 2\log_b x$$

$$\begin{aligned}\log_{\sqrt{b}} x &= \frac{\log x}{\log \sqrt{b}} \\ &= \frac{\log x}{\frac{1}{2}\log b} \\ &= 2\frac{\log x}{\log b} \\ &= 2\log_b x \quad \square\end{aligned}$$

(2)

$$\log_{\frac{1}{\sqrt{b}}} \sqrt{x} = -\log_b x$$

$$\begin{aligned}\log_{\frac{1}{\sqrt{b}}} \sqrt{x} &= \frac{\log \sqrt{x}}{\log \frac{1}{\sqrt{b}}} \\ &= \frac{\frac{1}{2}\log x}{-\frac{1}{2}\log b} \\ &= -\frac{\log x}{\log b} \\ &= -\log_b x \quad \square\end{aligned}$$

(3)

$$\log_{b^4} x^2 = \log_b \sqrt{x}$$

$$\begin{aligned}\log_{b^4} x^2 &= \frac{\log x^2}{\log b^4} \\ &= \frac{2\log x}{4\log b} \\ &= \frac{1}{2}\frac{\log x}{\log b} \\ &= \frac{1}{2}\log_b x \\ &= \log_b \sqrt{x} \quad \square\end{aligned}$$

9. (1) $2x + y$

(2) $x + 2$

(3) $x - y + z$

(4) $y - 1$

(5) $y - x$

(6) $y + z - x$

(7) $1 - x + y$

(8) $x + y - z + 3$

10. (1) $S = \{2.402\}$

(2) $S = \{0.369\}$

(3) $S = \{-7.213\}$

(4) $S = \{0.438\}$

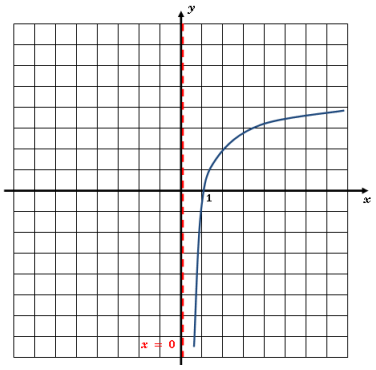
(5) $S = \{-1.652\}$

(6) $S = \{-\ln 4\}$

(7) $S = \{\log_5 4\}$

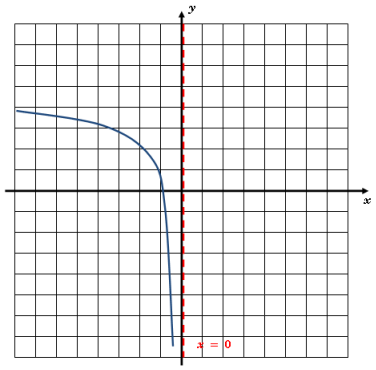
(8) $S = \{\ln 5\}$

11. (1)



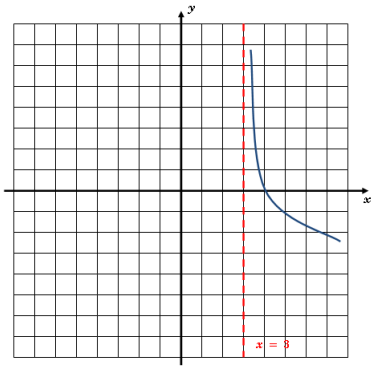
$\mathrm{Dom}(f) =]0, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: 1
Y-intercept: None
Variation:
$f(x) \nearrow$ if $x \in]0, +\infty[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]0, 1]$
$f(x) \leq 0$ if $x \in [1, +\infty[$

(2)



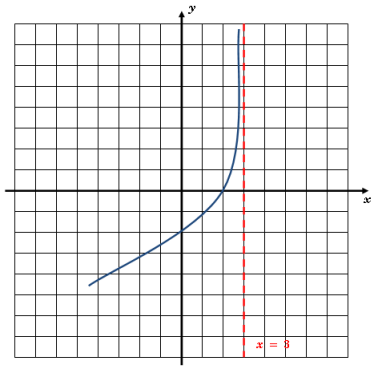
$\mathrm{Dom}(f) =]-\infty, 0[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: -1
Y-intercept: None
Variation:
$f(x) \nearrow$ if $x \in \emptyset$
$f(x) \searrow$ if $x \in]-\infty, 0[$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]-\infty, -1]$
$f(x) \leq 0$ if $x \in [-1, 0[$

(3)



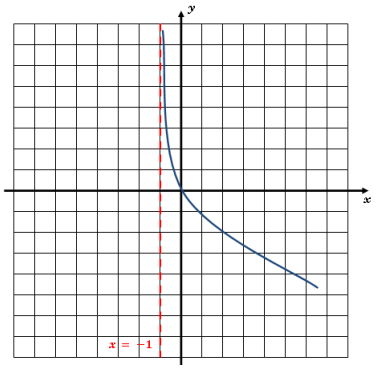
$\mathrm{Dom}(f) =]3, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: 4
Y-intercept: None
Variation:
$f(x) \nearrow$ if $x \in \emptyset$
$f(x) \searrow$ if $x \in]3, +\infty[$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]3, 4]$
$f(x) \leq 0$ if $x \in [4, +\infty[$

(4)



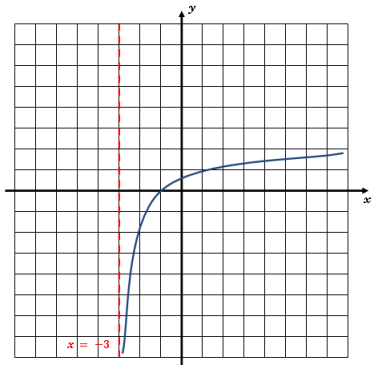
$\mathrm{Dom}(f) =]-\infty, 3[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: 2
Y-intercept: -2
Variation:
$f(x) \nearrow$ if $x \in]-\infty, 3[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]2, 3[$
$f(x) \leq 0$ if $x \in]-\infty, 2[$

(5)



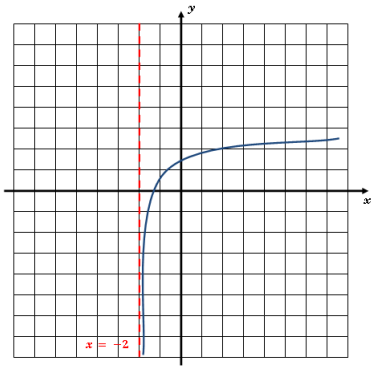
$\mathrm{Dom}(f) =]-1, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: 0
Y-intercept: 0
Variation:
$f(x) \nearrow$ if $x \in \emptyset$
$f(x) \searrow$ if $x \in]-1, +\infty[$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]-1, 0[$
$f(x) \leq 0$ if $x \in]0, +\infty[$

(6)



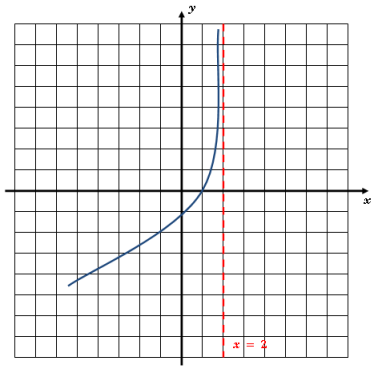
$\mathrm{Dom}(f) =]-3, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: -1
Y-intercept: $2 \ln \dfrac{3}{2}$
Variation:
$f(x) \nearrow$ if $x \in]-3, +\infty[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in [-1, +\infty[$
$f(x) \leq 0$ if $x \in]-3, -1]$

(7)



$\mathrm{Dom}(f) =]-2, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: -1.5
Y-intercept: $\ln 4$
Variation:
$f(x) \nearrow$ if $x \in]-2, +\infty[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in [-1.5, +\infty[$
$f(x) \leq 0$ if $x \in]-2, -1.5]$

(8)



$\mathrm{Dom}(f) =]-\infty, 2[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: $\dfrac{5}{3}$
Y-intercept: $-2\ln 6$
Variation:
$f(x) \nearrow$ if $x \in]-\infty, 2[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in [\frac{5}{3}, 2[$
$f(x) \leq 0$ if $x \in]-\infty, \frac{5}{3}[$

12. (1) $f^{-1}(x) = 2^{x+5} + 3$

    (2) $f^{-1}(x) = 3^{\frac{x-1}{3}} - 3$

    (3) $f^{-1}(x) = \frac{1}{2}10^{\frac{2-x}{2}} + 1$

    (4) $f^{-1}(x) = -\frac{1}{2}e^{1-x} + \frac{1}{2}$

    (5) $f^{-1}(x) = \log_2(x+3)$

    (6) $f^{-1}(x) = \frac{1}{3}\log_3\left(\frac{x+1}{2}\right)$

    (7) $f^{-1}(x) = -\ln\left(\frac{2-x}{5}\right)$

    (8) $f^{-1}(x) = -\frac{1}{2}\ln\left(\frac{1-x}{2}\right)$

13. 37 years.

14. 9 years.

15. 53 years old.

16. (a) $f(t) = 10000 \cdot 2^{1.5t}$. Where t is the number of hours.

    (b) 28 284 bacteria.

    (c) 92.88 minutes.