

$$01. \int_0^{2024} 2024 \, dx = \left(2024x \right) \Big|_0^{2024} = \boxed{2024^2}$$

$$02. \int_0^{2024} \underbrace{x+x+x+\dots+x}_{2024 \text{ vezes}} \, dx = \int_0^{2024} 2024x \, dx = \left(\frac{2024x^2}{2} \right) \Big|_0^{2024} = \boxed{\frac{2024^3}{2}}$$

$$03. \int \underbrace{(2024x^{2023} - 2023)}_{du} \cdot \underbrace{\cos(x^{2024} - 2023x)}_u \, dx = \sin u + C = \sin(x^{2024} - 2023x) + C$$

$$04. \int 2024 \ln x \, dx = 2024 \cdot \left(x \cdot \ln x - \int x \cdot \frac{1}{x} \, dx \right) = \boxed{2024x \cdot (\ln x - 1) + C}$$

$$05. \int (x^2 e^x + 2024) \, dx = x^2 e^x - \int 2x \cdot e^x \, dx + 2024x = x^2 e^x - 2(x \cdot e^x - e^x) + 2024x + C = \boxed{e^x(x^2 - 2x + 2) + 2024x + C}$$

$$06. \int \underbrace{(-x+2x-3x+4x-5x+\dots-2023x+2024x)}_x \, dx = \int 1012x \, dx = \boxed{506x^2 + C}$$

$$07. \int_0^{4048} \left[\underbrace{(x-2024)}_u^{2021} + \underbrace{(x-2024)}_u^{2022} + \underbrace{(x-2024)}_u^{2023} \right] \frac{dx}{du} = \int_{-2024}^{2024} \underbrace{\left(u^{2021} + u^{2022} + u^{2023} \right)}_{\text{ÍMPAR}} \, du$$

$$08. \int \frac{\sin^2(\ln x)}{2024x} \, dx = \frac{1}{2024} \int \underbrace{\sin^2 u}_{\frac{1}{2} - \frac{1}{2}\cos 2u} \, du = \int_{-2024}^{2024} u^{2022} \, du = \frac{2 \cdot 2024^{2023}}{2023}$$

$$09. \int \frac{x-2021}{(x-2022)(x-2023)(x-2024)} \, dx \rightarrow \frac{1}{2024} \left(\frac{u}{2} - \frac{\sin 2u}{4} \right) + C = \frac{1}{2024} \left(\frac{\ln x}{2} - \frac{\sin(2 \cdot \ln x)}{4} \right) + C$$

$$10. \int_1^2 \left(\sum_{i=1}^{2024} \frac{1}{(x+i-1)(x+i)} \right) dx \rightarrow \left(\frac{1}{\frac{1}{2}} \ln|x-2022| + \frac{1}{-2} \ln|x-2023| + \frac{1}{\frac{3}{2}} \ln|x-2024| \right) + C$$

$$= \int_1^2 \left(\frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)} + \dots + \frac{1}{(x+2023)(x+2024)} \right) dx = \int_1^2 \left(\frac{1}{x} - \frac{1}{x+2024} \right) dx$$

$$= \boxed{\ln 2 + \ln 2025 - \ln 2026}$$

